Effects of the $U_A(1)$ Anomaly on $\eta \to 2\gamma$ Decay

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Abstract

We study the $\eta \to 2\gamma$ decay using an extended three-flavor Nambu-Jona-Lasinio model that includes the 't Hooft instanton induced interaction. The η meson mass and the $\eta \to 2\gamma$ decay width are reproduced simultaneously with a rather strong instanton induced interaction. The calculated η decay constant is $f_\eta = 2.23 f_\pi$ and it suggests that the η meson is no longer the Goldstone boson.

I. INTRODUCTION

The structure of the η meson gives us information about the mechanisms of spontaneous breaking of chiral symmetry, the pattern of the explicit breaking of chiral symmetry and the dynamics of the $U_A(1)$ anomaly. The η meson is the eighth member of the low-lying nonet pseudoscalar mesons and is considered as a Goldstone boson associated with the spontaneous breaking of chiral symmetry in the QCD vacuum.

The physics of the η and η' mesons have been extensively studied in the $1/N_C$ expansion approach [1]. In the $N_C \to \infty$ limit, the $U_A(1)$ anomaly is turned off and then the η meson becomes to degenerate with the pion and the η' meson becomes pure $\bar{s}s$ state with $m_{\eta'}^2(N_C \to \infty) = 2m_K^2 - m_\pi^2 \simeq (687 \, \text{MeV})^2$ [2]. So the $U_A(1)$ anomaly pushes up m_η by about

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400MeV and $m_{\eta'}$ by about 300MeV. Usually the η' meson is not considered as a Goldstone boson since it is considered as mostly flavor singlet meson and in this channel there exists the $U_A(1)$ anomaly which explicitly breaks the chiral symmetry. On the other hand, the η meson is considered as a Goldstone boson because its mass is close to the naive estimation of the pseudoscalar meson mass in λ^8 channel, i.e., $m_{\eta_8}^2 = \frac{4}{3}m_K^2 - \frac{1}{3}m_\pi^2 \simeq (567\,\mathrm{MeV})^2$. From the $1/N_C$ expansion point of view, η-meson mass is largely affected by the finite N_C correction, i.e., the $U_A(1)$ anomaly, then it is natural to ask how much η meson loses the Goldstone boson nature.

In order to answer to the above question, it may be important to study the $\eta \to 2\gamma$ decay because of relation to the Adler-Bell-Jackiw (ABJ) triangle anomaly [3] through the partial conservation of axial-vector current (PCAC) hypothesis.

One of the useful and widely used frameworks for studying the phenomena related to the axial-vector anomaly and the spontaneous chiral symmetry breaking is the chiral effective meson lagrangian given by Wess and Zumino [4] and developed by Witten [5]. The $\eta, \eta' \to 2\gamma$ decays have been studied using the Wess-Zumino-Witten (WZW) lagrangian with the corrections at one-loop order in the chiral perturbation and it has been shown that the two-photon decay widths can be explained with the η-η' mixing angle $\theta \simeq -20°$ [6]. From the chiral perturbation [7] point of view, the WZW term is derived in the chiral limit and is of order p^4. As discussed in [8], to reliably calculate $SU(3)$ breaking effects of the $\eta, \eta' \to 2\gamma$ decays, the low-energy expansion to order p^6 has to be carried out. However in [6] full analysis of order p^6 has not been performed. Furthermore, because of the $U_A(1)$ anomaly, the singlet channel decay amplitude $\eta_0 \to 2\gamma$ derived using PCAC + ABJ anomaly should be modified so as to become the renormalisation group invariant [9].

The purpose of this paper is to study the $\eta \to 2\gamma$ decay in the framework of the generalized Nambu-Jona-Lasinio (NJL) model [10] as a chiral effective quark lagrangian of the low-energy QCD. The generalized three-flavor NJL model which involves the $U_L(3) \times U_R(3)$ symmetric four-quark interaction and the six-quark flavor-determinant interaction [11] incorporating effects of the $U_A(1)$ anomaly is used widely in recent years to study such topics as the quark condensates in vacuum, the spectrum of low-lying mesons, the flavor-mixing properties of the low-energy hadrons, etc. [12–16]. In this approach the effects of the explicit breaking of the chiral symmetry by the current quark mass term and the $U_A(1)$ anomaly on the $\eta \to 2\gamma$ decay amplitude can be calculated consistently with those on the η-meson mass, η decay constant and mixing angle within the model applicability.

II. EXTENDED NAMBU-JONA-LASINIO MODEL

We work with the NJL model lagrangian density extended to three-flavor case:

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_4 + \mathcal{L}_6, \tag{1}$$

$$\mathcal{L}_0 = \bar{\psi} \left(i\partial_\mu \gamma^\mu - \hat{m} \right) \psi, \tag{2}$$

$$\mathcal{L}_4 = \frac{G_S}{2} \sum_{a=0}^{8} \left[\left(\bar{\psi}\lambda^a\psi \right)^2 + \left(\bar{\psi}\lambda^a i\gamma_5\psi \right)^2 \right], \tag{3}$$

$$\mathcal{L}_6 = G_D \left\{ \det \left[\bar{\psi}_i(1-\gamma_5)\psi_j \right] + \det \left[\bar{\psi}_i(1+\gamma_5)\psi_j \right] \right\}. \tag{4}$$

Here the quark field ψ is a column vector in color, flavor and Dirac spaces and λ^a is the $U(3)$ generator in flavor space. The free Dirac lagrangian $\mathcal{L}_0$ incorporates the current quark mass matrix $\hat{m} = \mathrm{diag}(m_u, m_d, m_s)$ which breaks the chiral $U_L(3) \times U_R(3)$ invariance explicitly. $\mathcal{L}_4$ is a QCD motivated four-fermion interaction, which is chiral $U_L(3) \times U_R(3)$ invariant. The 't Hooft determinant $\mathcal{L}_6$ represents the $U_A(1)$ anomaly. It is a 3×3 determinant with respect to flavor with $i, j = \mathrm{u}, \mathrm{d}, \mathrm{s}$.

Quark condensates and constituent quark masses are self-consistently determined by the gap equations

$$
\begin{aligned}
M_u &= m_u - 2G_S\langle \bar{u}u \rangle - 2G_D\langle \bar{d}d \rangle\langle \bar{s}s \rangle, \\
M_d &= m_d - 2G_S\langle \bar{d}d \rangle - 2G_D\langle \bar{s}s \rangle\langle \bar{u}u \rangle, \\
M_s &= m_s - 2G_S\langle \bar{s}s \rangle - 2G_D\langle \bar{u}u \rangle\langle \bar{d}d \rangle,
\end{aligned}
\tag{5}
$$

with

$$\langle \bar{a}a \rangle = -\mathrm{Tr}^{(c,D)} \left[iS_F^a(x=0) \right] = -\int^{\Lambda} \frac{d^4p}{(2\pi)^4} \mathrm{Tr}^{(c,D)} \left[\frac{i}{p_\mu\gamma^\mu - M_a + i\varepsilon} \right]. \tag{6}$$

Here the covariant cutoff Λ is introduced to regularize the divergent integral and $\mathrm{Tr}^{(c,D)}$ means trace in color and Dirac spaces.

The pseudoscalar channel quark-antiquark scattering amplitudes $\langle p_3, \bar{p}_4; \mathrm{out}|p_1, \bar{p}_2; \mathrm{in} \rangle = (2\pi)^4 \delta^4(p_3 + p_4 - p_1 - p_2)\mathcal{T}_{q\bar{q}}$ are then calculated in the ladder approximation. We assume the isospin symmetry too. In the η and η' channel, the explicit expression is

$$\mathcal{T}_{q\bar{q}} = - \begin{pmatrix} \bar{u}(p_3)\lambda^8 i\gamma_5 v(p_4) \\ \bar{u}(p_3)\lambda^0 i\gamma_5 v(p_4) \end{pmatrix}^T \begin{pmatrix} A(q^2) & B(q^2) \\ B(q^2) & C(q^2) \end{pmatrix} \begin{pmatrix} \bar{v}(p_2)\lambda^8 i\gamma_5 u(p_1) \\ \bar{v}(p_2)\lambda^0 i\gamma_5 u(p_1) \end{pmatrix}, \tag{7}$$

with

$$A(q^2) = \frac{2}{\det \mathbf{D}(q^2)} \left\{ 2(G_0 G_8 - G_m G_m) I^0(q^2) - G_8 \right\}, \tag{8}$$

$$B(q^2) = \frac{2}{\det \mathbf{D}(q^2)} \left\{ -2(G_0 G_8 - G_m G_m) I^m(q^2) - G_m \right\}, \tag{9}$$

$$C(q^2) = \frac{2}{\det \mathbf{D}(q^2)} \left\{ 2(G_0 G_8 - G_m G_m) I^8(q^2) - G_0 \right\}, \tag{10}$$

and $G_0 = \frac{1}{2} G_S - \frac{1}{3}(2\langle \bar{u}u \rangle + \langle \bar{s}s \rangle) G_D$, $G_8 = \frac{1}{2} G_S - \frac{1}{6}(\langle \bar{s}s \rangle - 4\langle \bar{u}u \rangle) G_D$, $G_m = -\frac{1}{3\sqrt{2}}(\langle \bar{s}s \rangle - \langle \bar{u}u \rangle) G_D$. The quark-antiquark bubble integrals are

$$I^0(q^2) = i \int^{\Lambda} \frac{d^4 p}{(2\pi)^4} \mathrm{Tr}^{(c,f,D)} \left[S_F(p) \lambda^0 i\gamma_5 S_F(p+q) \lambda^0 i\gamma_5 \right], \tag{11}$$

$$I^8(q^2) = i \int^{\Lambda} \frac{d^4 p}{(2\pi)^4} \mathrm{Tr}^{(c,f,D)} \left[S_F(p) \lambda^8 i\gamma_5 S_F(p+q) \lambda^8 i\gamma_5 \right], \tag{12}$$

$$I^m(q^2) = i \int^{\Lambda} \frac{d^4 p}{(2\pi)^4} \mathrm{Tr}^{(c,f,D)} \left[S_F(p) \lambda^0 i\gamma_5 S_F(p+q) \lambda^8 i\gamma_5 \right], \tag{13}$$

with $q = p_1 + p_2$. The 2×2 matrix $\mathbf{D}$ is

$$\mathbf{D}(q^2) = \begin{pmatrix} D_{11}(q^2) & D_{12}(q^2) \\ D_{21}(q^2) & D_{22}(q^2) \end{pmatrix}, \tag{14}$$

with

$$D_{11}(q^2) = 2G_8 I^8(q^2) + 2G_m I^m(q^2) - 1, \tag{15}$$

$$D_{12}(q^2) = 2G_8 I^m(q^2) + 2G_m I^0(q^2) \tag{16}$$

$$D_{21}(q^2) = 2G_0 I^m(q^2) + 2G_m I^8(q^2) \tag{17}$$

$$D_{22}(q^2) = 2G_0 I^0(q^2) + 2G_m I^m(q^2) - 1. \tag{18}$$

From the pole position of the scattering amplitude Eq. (7), the η-meson mass m_η is determined.

The scattering amplitude Eq. (7) can be diagonalized by rotation in the flavor space

$$\mathcal{T}_{q\bar{q}} = - \begin{pmatrix} \bar{u}(p_3) \lambda^8 i\gamma_5 v(p_4) \\ \bar{u}(p_3) \lambda^0 i\gamma_5 v(p_4) \end{pmatrix}^T \mathbf{T}_\theta^{-1} \mathbf{T}_\theta \begin{pmatrix} A(q^2) & B(q^2) \\ B(q^2) & C(q^2) \end{pmatrix} \mathbf{T}_\theta^{-1} \mathbf{T}_\theta \begin{pmatrix} \bar{v}(p_2) \lambda^8 i\gamma_5 u(p_1) \\ \bar{v}(p_2) \lambda^0 i\gamma_5 u(p_1) \end{pmatrix}, \tag{19}$$

$$= - \begin{pmatrix} \bar{u}(p_3) \lambda^\eta i\gamma_5 v(p_4) \\ \bar{u}(p_3) \lambda^{\eta'} i\gamma_5 v(p_4) \end{pmatrix}^T \begin{pmatrix} D^\eta(q^2) & 0 \\ 0 & D^{\eta'}(q^2) \end{pmatrix} \begin{pmatrix} \bar{v}(p_2) \lambda^\eta i\gamma_5 u(p_1) \\ \bar{v}(p_2) \lambda^{\eta'} i\gamma_5 u(p_1) \end{pmatrix}, \tag{20}$$

with $\lambda^\eta \equiv \cos\theta \lambda^8 - \sin\theta \lambda^0$, $\lambda^{\eta'} \equiv \sin\theta \lambda^8 + \cos\theta \lambda^0$ and

$$\mathbf{T}_\theta = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}.$$

The rotation angle θ is determined by

$$\tan 2\theta = \frac{2B(q^2)}{C(q^2) - A(q^2)}. \tag{21}$$

So θ depends on q^2. At $q^2 = m_\eta^2$, θ represents the mixing angle of the λ^8 and λ^0 components in the η-meson state. In the usual effective pseudoscalar meson lagrangian approaches, the η and η' mesons are analyzed using the q^2-independent η-η' mixing angle. Because of the q^2-dependence, θ cannot be interpreted as the η-η' mixing angle. The origin of the q^2-dependence is that the η and η' meson have the internal structures.

The effective η-quark coupling constant g_η is determined by the residue of the scattering amplitude at the η pole, i.e., $g_\eta^2 = \lim_{q^2 \to m_\eta^2}(q^2 - m_\eta^2)D_\eta(q^2)$, and the η decay constant f_η is determined by calculating the quark-antiquark one-loop graph,

$$f_\eta = \frac{g_\eta}{m_\eta^2} \int^\Lambda \frac{d^4p}{(2\pi)^4} \mathrm{Tr}^{(c,f,D)}[q^\mu \gamma_\mu \gamma_5 \frac{\lambda^\eta}{2} S_F(p) i\gamma_5 \lambda^\eta S_F(p-q)]\Big|_{q^2=m_\eta^2}. \tag{22}$$

One can easily show that in the $U_A(1)$ limit, i.e., $G_D = 0$ and $m_{u,d} \neq m_s$, the η meson becomes the ideal mixing state composed of u and d-quarks, namely, $m_\eta = m_\pi$, $g_\eta = g_\pi$, $f_\eta = f_\pi$ and $\tan\theta = -\sqrt{2}$.

III. $\eta \to 2\gamma$ DECAY AMPLITUDES

Let us now turn to the calculations of the $\pi^0, \eta \to 2\gamma$ decay widths. The $\pi^0, \eta \to 2\gamma$ decay amplitudes are given by

$$\langle \gamma(k_1)\gamma(k_2)|M(q)\rangle = i(2\pi)^4 \delta^4(k_1 + k_2 - q)\varepsilon_{\mu\nu\rho\sigma}\epsilon_1^\mu \epsilon_2^\nu k_1^\rho k_2^\sigma \widetilde{\mathcal{T}}_{M\to 2\gamma}(q^2), \tag{23}$$

where ϵ_1 and ϵ_2 are the polarization vectors of the photon. By calculating the pseudoscalar-vector-vector type quark triangle diagrams, we get the following results.

$$\widetilde{\mathcal{T}}_{\pi^0 \to 2\gamma} = \frac{\alpha}{\pi} g_\pi F(u, \pi^0), \tag{24}$$

$$\widetilde{\mathcal{T}}_{\eta \to 2\gamma} = \frac{\alpha}{\pi} g_\eta \frac{1}{3\sqrt{3}} \left[\cos\theta \left\{5F(u,\eta) - 2F(s,\eta)\right\} - \sin\theta\sqrt{2}\left\{5F(u,\eta) + F(s,\eta)\right\}\right]. \tag{25}$$

Here α is the fine structure constant of QED and $F(a, M)$ ($a = u, s$ and $M = \pi^0, \eta$) is defined as

$$F(a, M) = \int_0^1 dx \int_0^1 dy \frac{2(1-x)M_a}{M_a^2 - m_M^2 x(1-x)(1-y)}. \tag{26}$$

Then the $M \to 2\gamma$ decay width $\Gamma(M \to 2\gamma)$ is given by $\Gamma(M \to 2\gamma) = |\widetilde{\mathcal{T}}_{M\to 2\gamma}|^2 m_M^3/(64\pi)$.

In the chiral limit, the pion mass vanishes and $F(u, \pi^0)$ becomes $1/M_u$. In this limit, the Goldberger-Treiman (GT) relation at the quark level, $M_u = g_\pi f_\pi$, holds in the NJL model and this leads to $\widetilde{\mathcal{T}}_{\pi^0 \to 2\gamma} = \alpha/(\pi f_\pi)$ which is same as the tree-level results in the WZW lagrangian approach. It should be mentioned that we have to integrate out the triangle diagrams without introducing a cutoff Λ in order to get the above result though the cutoff is introduced in Eqs. (6,11-13,22) in the NJL model. If we introduce a cutoff Λ to the loop-integral of the triangle diagrams, the decay amplitude becomes too small and we lose the success of the model independent prediction for the $\Gamma(\pi^0 \to 2\gamma)$. In the $U(3)_L \times U(3)_R$ version of the NJL model, the WZW term has been derived using the bosonization method with the heat-kernel expansion [17,18]. In thier approach, $O(1/\Lambda)$ term has been neglected and it is same as to take the $\Lambda \to \infty$ limit.

IV. NUMERICAL RESULTS

The recent experimental results of the $\pi^0, \eta \to 2\gamma$ decay widths are $\Gamma(\pi^0 \to 2\gamma) = 7.7 \pm 0.6\,\text{eV}$ and $\Gamma(\eta \to 2\gamma) = 0.510 \pm 0.026\,\text{keV}$ [19] and the reduced amplitudes are

$$\left|\widetilde{\mathcal{T}}_{\pi^0 \to 2\gamma}\right| = (2.5 \pm 0.1) \times 10^{-11}\,[\text{eV}]^{-1}, \tag{27}$$

$$\left|\widetilde{\mathcal{T}}_{\eta \to 2\gamma}\right| = (2.5 \pm 0.06) \times 10^{-11}\,[\text{eV}]^{-1}. \tag{28}$$

From Eq. (24) and Eq. (25), we get $\widetilde{\mathcal{T}}_{\eta \to 2\gamma} = (5/3)\widetilde{\mathcal{T}}_{\pi^0 \to 2\gamma}$ in the $U_A(1)$ limit. Therefore in order to reproduce the experimental value of $\widetilde{\mathcal{T}}_{\eta \to 2\gamma}$, the effect of the $U_A(1)$ anomaly should reduce $\widetilde{\mathcal{T}}_{\eta \to 2\gamma}$ by a factor 3/5.

In our theoretical calculations, the parameters of the NJL model are the current quark masses $m_u = m_d$, m_s, the four-quark coupling constant G_S, the coupling constant of the 't Hooft instanton induced interaction G_D and the covariant cutoff Λ. First, we take G_D as a free parameter and study the η-meson properties as functions of G_D. For the light quark masses, $m_u = m_d = 8.0$ MeV is taken to reproduce $M_u = M_d \simeq 330$ MeV which is the value usually used in the nonrelativistic quark model. Other parameters m_s, G_D and Λ are determined so as to reproduce the isospin averaged observed masses, $m_\pi = 138.04$ MeV, $m_K = 495.7$ MeV and the pion decay constant f_π.

The calculated constituent u,d-quark mass is $M_{u,d} = 324.9$ MeV which is independent of G_D. On the other hand the calculated constituent s-quark mass weakly decreases from $M_s = 556.3$ MeV to $M_s = 503.3$ MeV when G_D is changed from $G_D = 0$ to $G_D = G_D^\eta$ where the observed m_η is reproduced. The fitted result of the current s-quark mass m_s is almost independent of G_D and $m_s = 192.95$ MeV at $G_D = G_D^\eta$. The ratio of the current s-quark mass to the current u,d-quark mass is $m_s/m_u = 24.1$, which agrees well with $m_s/\hat{m} = 25 \pm 2.5$

$(\hat{m} = \frac{1}{2}(m_u + m_d))$ derived from ChPT [20]. The kaon decay constant f_K is the prediction and is almost independent of G_D. We have obtained $f_K = 96.6$ MeV at $G_D = G_D^\eta$ which is about 15% smaller than the observed value. We consider this is the typical predictive power of the NJL model in the strangeness sector.

We next discuss the $\pi^0 \to 2\gamma$ decay. The calculated result is $\widetilde{\mathcal{T}}_{\pi^0 \to 2\gamma} = 2.50 \times 10^{-11} 1/\text{eV}$ which agrees well with the observed value given in Eq. (27). The current algebra result is $\widetilde{\mathcal{T}}_{\pi^0 \to 2\gamma} = \alpha/(\pi f_\pi) = 2.514 \times 10^{-11} 1/\text{eV}$, so the soft pion limit is a good approximation for $\pi^0 \to 2\gamma$ decay. There are two effects of the symmetry breaking on $\widetilde{\mathcal{T}}_{\pi^0 \to 2\gamma}$. One is the deviation from the G-T relation and another is the matrix element of the triangle diagram $F(u, \pi^0)$. Our numerical results are $g_\pi = 3.44$, $M_u/f_\pi = 3.52$ and $F(u, \pi^0)M_u = 1.015$, therefore the deviations from the soft pion limit are very small both in the G-T relation and the matrix element of the triangle diagram.

Let us now turn to the discussion of the η-meson properties. The calculated results of the η-meson mass m_η and the mixing angle θ are shown in Fig. 1, the η decay constant f_η is given in Fig. 2 and the $\eta \to 2\gamma$ decay amplitude $\widetilde{\mathcal{T}}_{\eta \to 2\gamma}$ is given in Fig. 3 as functions of the non-dimensionalized coupling constant of 't Hooft instanton induced interaction $G_D^{eff} \equiv -G_D(\Lambda/2\pi)^4 \Lambda N_C^2$.

As can be seen from Fig. 1, in order to reproduce the observed η-meson mass, rather strong instanton induced interaction is necessary. For example, at $G_D = G_D^\eta$, $G_D \langle \bar{s}s \rangle / G_s = 1.58$, it means that the contribution of $\mathcal{L}_6$ to the dynamical mass of the u,d-quarks is about 60% bigger than that of $\mathcal{L}_4$. In the previous study of the η and η' mesons in the extended NJL model [12–16], the strength of the instanton induced interaction has been determined so as to reproduce the observed η' mass though the η' state has the unphysical decay mode of the $\eta' \to \bar{u}u, \bar{d}d$. The strength determined from $m_{\eta'}$, $G_D^{\eta'}$ is much smaller than G_D^η, about 1/10 to 1/5 of G_D^η. One of the shortcomings of the NJL model is the lack of the confinement mechanism. It is expected that the confinement gives rise to the attractive force between quark and antiquark in the η' meson to prevent the η' meson from decaying to the quark and antiquark pair.

One of the important results is that the calculated η decay constant is very different from the pion decay constant, $f_\eta = 206\,\text{MeV} = 2.23 f_\pi$ at $G_D = G_D^\eta$. This suggests that the η meson loses the Goldstone boson nature very much. To see whether the η meson is the Goldstone boson, the G-T relation is another important information. For the η meson, the naive G-T relation at the quark level is $2g_\eta f_\eta = |\frac{2}{\sqrt{3}}\cos\theta - 2\sqrt{\frac{2}{3}}\sin\theta|M_u + |-\frac{2}{\sqrt{3}}\cos\theta - \sqrt{\frac{2}{3}}\sin\theta|M_s$ and at $G_D = G_D^\eta$, our numerical results are $2g_\eta f_\eta = 3.202$ GeV and $|\frac{2}{\sqrt{3}}\cos\theta - 2\sqrt{\frac{2}{3}}\sin\theta|M_u + |-\frac{2}{\sqrt{3}}\cos\theta - \sqrt{\frac{2}{3}}\sin\theta|M_s = 0.892$ GeV. Therefore the G-T relation dose not hold at all. Since the NJL model is known as the model that describes the

Goldstone boson properties reasonably well, it is natural to ask whether the present model is applicable to the η meson. For the η' meson, we expect that the confinement plays an important role since the η'-meson pole of the scattering amplitude Eq. (7) appears above the $\bar{u}u$ and $\bar{d}d$-thresold. On the other hand, the η meson is the tight bound state, so we expect that the NJL model can describe the essential feature of the η meson. In order to confirm it, we have studied the constituent u,d-quark mass dependence of the η-meson properties. By changing $m_{u,d}$ from 7.5 MeV to 8.5 MeV, we have changed $M_{u,d}$ from about 300 MeV to 360 MeV and other parameters of the model have been chosen so as to reproduce the experimental values of m_π, m_K, m_η and f_π. The changes of the calculated η decay constant and the mixing angle have been within 2%.

We are now in the position to discuss the $\eta \to 2\gamma$ decay amplitude. Our result is $\widetilde{\mathcal{T}}_{\eta \to 2\gamma} = 2.73 \times 10^{-11}$ 1/eV at $G_D = G_D^\eta$, which is about 10% larger than the experimental value. Therefore the present model reproduces the η-meson mass and the $\eta \to 2\gamma$ decay width simultaneously. As for the effects of the symmetry breaking on $\widetilde{\mathcal{T}}_{\eta \to 2\gamma}$, our results are $F(u,\eta)M_u = 1.41$ and $F(u,\eta)/F(s,\eta) = 1.96$. Bernard et.al. [21] calculated the $\eta \to 2\gamma$ decay width using a similar model. They used a rather weak instanton induced interaction and their result of $\Gamma(\eta \to 2\gamma)$ is about 50% bigger than the experimental value. It is understandable from our analysis.

Our calculated result of the mixing angle is $\theta = 15.1°$ which should be compared with $\theta \simeq -20°$ [22]. There are two major differences between our model calculations and the usual analysis. One is that, in the usual analysis, the energy independent mixing is assumed. Another point is that the effects of the $U_A(1)$ anomaly on the η-meson properties are not taken into account in the usual analysis.

V. CONCLUDING REMARKS

Using an extended three-flavor Nambu-Jona-Lasinio model that includes the 't Hooft instanton induced interaction, we have studied the $\pi^0, \eta \to 2\gamma$ decays as well as the properties of the pion, the kaon and the η meson. The η-meson mass and the $\eta \to 2\gamma$ decay width have been reproduced simultaneously with a rather strong instanton induced interaction. The calculated η decay constant is about twice of the pion decay constant. So it is rather hard to consider the η meson as the Goldstone boson. Because of the above novel picture of the η meson, the situation of the mixing angle is also different from the usual analysis.

In order to confirm the novel picture of the η meson we have obtained here, certainly further studies of the η-meson properties are necessary. One thing is to study other η-meson decay processes such as $\eta \to \pi^0 2\gamma$, $\eta \to 3\pi$ in the present framework and such calculations

are now in progress. Since the properties of the η meson and those of the η' meson are closely related, one has to construct the low-energy effective model of QCD which can apply to the η' meson. Such an attempt is left as future study.

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Figure Captions

Fig. 1 Dependence of the calculated η-meson mass m_η (solid line) and the mixing angle θ (dashed line) on the non-dimensionalized coupling constant of the 't Hooft instanton induced interaction G_D^{eff}. The observed η-meson mass $m_\eta = 547.45$ MeV is reproduced at $G_D^{eff} = 1.41$.

Fig. 2 Dependence of the calculated η decay constant f_η on the non-dimensionalized coupling constant of the 't Hooft instanton induced interaction G_D^{eff}.

Fig. 3 Dependence of the $\eta \rightarrow 2\gamma$ decay amplitude $\widetilde{\mathcal{T}}_{\eta \rightarrow 2\gamma}$ on the non-dimensionalized coupling constant of the 't Hooft instanton induced interaction G_D^{eff}.